

Equation (3) is apparently not applicable to the case of diffusion of a low molecular weight liquid solute into a high molecular weight liquid solvent. For example, the measured limiting diffusion coefficient of benzene into natural rubber (Pattle et al., 1967) at 25°C is 1.37×10^{-7} cm²/s, whereas the value predicted by Equation (3) is 1.47×10^{-12} cm²/s, five orders of magnitude lower. Similar discrepancies were found for other low molecular weight solutes. In fact, the discrepancies in most cases amount to the same factor, that is, 10^5 . This aspect of the problem will be studied further.

ACKNOWLEDGMENT

This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract Number NAS 7-100 sponsored by the National Aeronautics and Space Administration.

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Manuscript received July 13, 1978; rebuttal received January 12, and accepted February 5, 1979.

Tracer Analysis in Systems with Two-Phase Flow

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Tracer analysis is an important tool in identification of many parameters in chemical engineering systems. The use of tracer impulse response techniques in single-phase systems leads to estimation of the active volume of the system, dead water regions, degree of backmixing, etc. (Levenspiel, 1972; Himmelblau and Bischoff, 1968; Wen and Fan, 1975). The same basic type of impulse or step response dynamic measurements can be applied to multiphase systems with only one flowing phase to estimate, besides the above-mentioned parameters, quantities such as fluid holdup, fluid-solid mass transfer coefficients, fluid-solid adsorption equilibrium constants, adsorption rate constants, surface kinetic constants, diffusivities in porous packings, etc. (Masamune and Smith, 1964; Suzuki and Smith, 1971; Misic and Smith, 1971; Haynes and Sarma, 1973; Haynes, 1975). Recently, the use of tracer dynamic measurements has been extended to systems with multiphase flow to evaluate fluid-fluid partition coefficients, holdup of each of the flowing phases, transport coefficients, effective areas of stagnant packing contacted by one of the flowing phases, degree of backmixing, etc. (Perilloux and Deans, 1972; Schwartz et al., 1976). The theory of tracer experiments has been extended to cover three-phase systems with two-phase flow, such as slurry reactors (Ramachandran and Smith, 1977) and a two-phase representation of the fluidized-bed reactor (Dayan and Levenspiel, 1970).

The interpretation of tracer dynamic experiments often relies on the analysis of moments of the response curve

(Levenspiel, 1972; Suzuki and Smith, 1971). The advantages and disadvantages of this type of data analysis and comparison to other methods have been presented (Seinfeld and Lapidus, 1974). In spite of the possible pitfalls associated with the method of moments, the use of the technique is widely utilized owing to its simplicity. It is, therefore, of interest to evaluate the moments for the models of the system and by comparison of these expressions with experimentally calculated moments extract the values of desired system parameters.

The objective of this note is to outline the main differences in moments interpretation in single-phase and two-phase flow systems and to point out some results unique to the two-phase flow systems. Of particular interest is the effect of tracer volatility on the moments which was first observed and reported by Schwartz et al. (1976). The equations are presented for two-phase concurrent and countercurrent flow in packed beds.

DEVELOPMENT

A packed bed with concurrent or countercurrent flow is considered. Both phases are assumed to be in plug flow, which is a reasonable approximation for most systems of practical interest. Let the two phases be liquid and gas, although the equations are applicable to two partially immiscible liquids also. The governing equations describing the response to an impulse injection of tracer are

$$H_L \frac{\partial C_L}{\partial t} = -u_L \frac{\partial C_L}{\partial z} - k_{LG} a_{LG} \left(C_L - \frac{C_G}{K_{LG}} \right)$$

$$-k_{LS}a_{LS}\left(C_L - \frac{C_S}{K_{LS}}\right) \quad (1)$$

$$H_G \frac{\partial C_G}{\partial t} = \mp u_G \frac{\partial C_G}{\partial z} + k_{LG}a_{LG}\left(C_L - \frac{C_G}{K_{LG}}\right) - k_{GS}a_{GS}\left(C_G - \frac{C_S}{K_{LS}}\right) \quad (2)$$

$$(1 - H_L - H_G) \frac{\partial C_S}{\partial t} = k_{LS}a_{LS}\left(C_L - \frac{C_S}{K_{LS}}\right) + k_{GS}a_{GS}\left(C_G - \frac{C_S}{K_{GS}}\right) \quad (3)$$

$$t = 0; \quad C_G = C_L = C_S = 0 \quad (4)$$

$$z = 0 \quad C_L(t, 0) = \frac{M_T}{Q_L} f \delta(t) \quad (5)$$

TABLE 1. LAPLACE TRANSFORM EXPRESSIONS FOR THE GAS AND LIQUID EXITING CONCENTRATIONS

A. Concurrent flow

1. Liquid phase

$$\frac{\bar{C}_L(s, L)}{C_{L,0}} = \frac{f_1[(\lambda_1 - a)e^{\lambda_2} - (\lambda_2 - a)e^{\lambda_1}] + f_2b(e^{\lambda_1} - e^{\lambda_2})}{[(a - d)^2 + 4\omega b^2]^{1/2}}$$

2. Gas phase

$$\frac{\bar{C}_G(s, L)}{C_{L,0}} = K_{LG} \left\{ \frac{f_1(\lambda_1 - a)(\lambda_2 - a)(e^{\lambda_2} - e^{\lambda_1}) + f_2b[(\lambda_1 - a)e^{\lambda_1} - (\lambda_2 - a)e^{\lambda_2}]}{b[(a - d)^2 + 4\omega b^2]^{1/2}} \right\}$$

B. Countercurrent flow

1. Liquid phase

$$\frac{\bar{C}_L(s, L)}{C_{L,0}} = \frac{f_1(\lambda_1' - \lambda_2')e^{\lambda_1' + \lambda_2'} + f_2b(e^{\lambda_1'} - e^{\lambda_2'})}{(\lambda_1' - a)e^{\lambda_1'} - (\lambda_2' - a)e^{\lambda_2'}}$$

2. Gas phase

$$\frac{\bar{C}_G(s, 0)}{C_{L,0}} = K_{LG} \left\{ \frac{f_1(\lambda_1' - a)(\lambda_2' - a)(e^{\lambda_1'} - e^{\lambda_2'}) + f_2b(\lambda_1' - \lambda_2')}{(\lambda_1' - a)e^{\lambda_1'} - (\lambda_2' - a)e^{\lambda_2'}} \right\}$$

where

$$a = \frac{\alpha_2^2 \delta}{s + (\alpha_2 + \alpha_3) \delta} - s - (\alpha_1 + \alpha_2)$$

$$b = \alpha_1 + \frac{\alpha_2 \alpha_3 \delta}{s + (\alpha_2 + \alpha_3) \delta}$$

$$d = \omega \left[\frac{\alpha_3^2 s}{s + (\alpha_2 + \alpha_3) \delta} - \frac{s}{\gamma} - (\alpha_1 + \alpha_3) \right]$$

$$\lambda_1 = \frac{a + d + [(a - d)^2 + 4\omega b^2]^{1/2}}{2};$$

$$\lambda_2 = \frac{a + d - [(a - d)^2 + 4\omega b^2]^{1/2}}{2}$$

$$\lambda_1' = \frac{a - d + [(a + d)^2 - 4\omega b^2]^{1/2}}{2};$$

$$\lambda_2' = \frac{a - d - [(a + d)^2 - 4\omega b^2]^{1/2}}{2}$$

$$C_G(t, 0) = \frac{M_T}{Q_G} (1 - f) \delta(t) \quad (6a)$$

or

$$z = L \quad C_G(t, L) = \frac{M_T}{Q_G} (1 - f) \delta(t) \quad (6b)$$

In deriving the above equations, it has been assumed that the tracer may distribute itself in both flowing phases and on the stagnant solid phase. The equations as written are for a bed of nonporous solid particles, but the results as far as the zeroth and first moment are concerned are also applicable to beds of porous solids. Complete equations accounting for diffusion into porous solid particles were presented by Schwartz (1975). The solid porosity and diffusion in the solid affect the expressions for higher moments only. For a nonvolatile tracer, $k_{LG} = k_{GS} = 0$, and the system is reduced to a familiar single-phase flow system with a stagnant phase as in adsorption columns. For a nonadsorbable tracer, $k_{LS} = k_{GS} = 0$, and tracer exchange occurs only between the flowing phases. The negative sign in front of the first term on the right side of Equation (2) and boundary conditions (5) and (6a) should be used for concurrent flow, while the positive sign in Equation (2) and Equations (5) and (6b) are valid for countercurrent flow. Parameter f indicates the fraction of the tracer that is injected into the liquid stream.

Equations (1) to (6) can be properly dimensionalized (Schwartz, 1975; Wu, 1978) and solved by using the Laplace transform in time, since in the transform domain they result in two-first-order ordinary differential equations and one algebraic equation. From the Laplace transform solution given in Table 1, expressions for the moments can be obtained by standard techniques (Seinfeld and Lapidus, 1974).

The relationship between the actual moments for the tracer concentration in the liquid $(\mu_n)_L$, or gas $(\mu_n)_G$

TABLE 2. EXPRESSIONS FOR THE ZEROth MOMENT FOR TWO-PHASE PLUG FLOW IN PACKED BEDS

1. Concurrent flow

$$(\mu_0^*)_L = \frac{f}{1 + \omega} [\omega + e^{-(1+\omega)K^*}] + \frac{\omega(1-f)}{1 + \omega} [1 - e^{-(1+\omega)K^*}]$$

$$(\mu_0^*)_G = \frac{\omega f}{1 + \omega} [1 - e^{-(1+\omega)K^*}] + \frac{\omega(1-f)}{1 + \omega} [1 + \omega e^{-(1+\omega)K^*}]$$

2. Countercurrent flow (for $f_1 = 1, f_2 = 0$ or $f_1 = 0, f_2 = 1$)

a) $\omega = 1$

$$(\mu_0^*)_L = f_1 \frac{1}{1 + K^*} + f_2 \frac{K^*}{1 + K^*}$$

$$(\mu_0^*)_G = f_1 \frac{K^*}{1 + K^*} + f_2 \frac{1}{1 + K^*}$$

b) $\omega \neq 1$

$$(\mu_0^*)_L = f_1 \frac{(\omega - 1)e^{(\omega-1)K^*}}{\omega e^{(\omega-1)K^*} - 1} + f_2 \frac{e^{(\omega-1)K^*} - 1}{\omega e^{(\omega-1)K^*} - 1}$$

$$(\mu_0^*)_G = f_1 \frac{\omega(e^{(\omega-1)K^*} - 1)}{\omega e^{(\omega-1)K^*} - 1} + f_2 \frac{\omega - 1}{\omega e^{(\omega-1)K^*} - 1}$$

and the moments for the dimensionless variables $(\mu_n^*)_L$, $(\mu_n^*)_G$ are given below:

$$(\mu_n)_L = \frac{M_T}{ALH_L} \left(\frac{LH_L}{u_L} \right)^{n+1} (\mu_n^*)_L \quad (7)$$

$$(\mu_n)_G = \frac{M_T K_{LG}}{ALH_L} \left(\frac{LH_L}{u_L} \right)^{n+1} (\mu_n^*)_G \quad (8)$$

The expressions for the zeroth moment are summarized in Table 2. It should be noted that the dimensionless zeroth moments must satisfy the overall mass balance expressed by Equation (9):

$$(\mu_0^*)_L + \frac{1}{\omega} (\mu_0^*)_G = 1 \quad (9)$$

The complete expressions for the first moment are even more cumbersome and are not reported here but are available upon request (Wu, 1978).

RESULTS AND DISCUSSION

The zeroth moments of the dynamic impulse response represent simultaneously the steady state effluent concentrations in a packed-bed adsorber with two-phase flow. This is the consequence of the nature of the Laplace transform, since it can clearly be seen that transforming Equations (1) to (6) and setting the transform variable s equal to zero results in the steady state expressions. As expected, the zeroth moment is a function of two parameters: the overall dimensionless mass transfer coefficient K^* and the ratio of liquid to gas tracer flow rate ω :

$$K^* = \alpha_1 + \frac{\alpha_2 \alpha_3}{\alpha_2 + \alpha_3} \quad (10a)$$

$$\omega = \frac{u_L}{u_G K_{LG}} \quad (10b)$$

$$\alpha_1 = \frac{k_{LG} a_{LG} L}{u_L} \quad (11a)$$

$$\alpha_2 = \frac{k_{LS} a_{LS} L}{u_L} \quad (11b)$$

$$\alpha_3 = \frac{k_{GS} a_{GS} L}{u_L} \quad (11c)$$

The dimensionless mass transfer coefficient encompasses the gas-liquid mass transfer step in series with the liquid-solid mass transfer step and is parallel with the gas-solid

mass transfer step. When either of the volumetric mass transfer coefficients for gas-solid or liquid-solid transfer is negligibly small, the dimensionless mass transfer coefficient K^* is reduced to α_1 , that is, the ratio of gas-liquid mass transfer rate to liquid flow rate.

The results for the zeroth moment in the liquid and gas effluent subject to an impulse injection in the liquid stream ($f = 1$) are presented in Figures 1 and 2. For concurrent flow (Figure 1), the zeroth moment for the liquid $(\mu_0^*)_L$ decreases monotonically with increasing K^* , while the zeroth moment for the gas stream $(\mu_0^*)_G$ increases. Both moments tend to an asymptotic value of $\omega/(1 + \omega)$ which also represents the zeroth moment in case of an equilibrated impulse into both phases. The higher the value of ω , the lower the value of K^* for which the asymptotic value is reached within negligible error. Opposite trends for the moments are found in case of an injection into the gas phase only ($f = 0$). Since ω is known, the measurement of the zeroth moments allows, in principle, determination of gas-liquid mass transfer coefficients provided $\alpha_1 \gg \alpha_2 \alpha_3 / (\alpha_2 + \alpha_3)$. Determination of gas-solid and liquid-solid mass transfer coefficients is also possible if gas-liquid coefficients are known.

In the case of countercurrent flow (Figure 2), the expressions for $(\mu_0^*)_L$ and $(\mu_0^*)_G$ are symmetric for $\omega = 1$ and cross over at $K^* = 1$. For $\omega > 1$, most of the tracer tends to come out with the liquid stream even at high mass transfer rates and as $K^* \rightarrow \infty$, $(\mu_0^*)_L \rightarrow 1 - 1/\omega$. The fraction of tracer leaving with the gas is proportional to $(\mu_0^*)_G/\omega$ and rises from 0 to $1/\omega$ at high K^* . For $\omega < 1$, the fraction of the tracer leaving with the liquid decays to zero at high K^* , and the fraction leaving with the gas rises asymptotically to one. This is the known behavior of such columns at steady state.

The results for the first moments subject to the additional assumption that gas-solid mass transfer is negligible ($k_{GS} a_{GS} \rightarrow 0$), which leads to $K^* \rightarrow \alpha_1$, are presented in Figures 3 and 4 for concurrent flow and impulse injection in the liquid ($f = 1$). The first moment is a function of several parameters: K^* , ω , β , and δ . The two additional parameters are the ratio of liquid and gas mean residence times β and the ratio of liquid capacity for the tracer to that of the solid phase δ :

$$\beta = \frac{u_G H_L}{u_L H_G} \quad (12a)$$

$$\delta = \frac{H_L}{(1 - H_G - H_L) K_{LS}} \quad (12b)$$

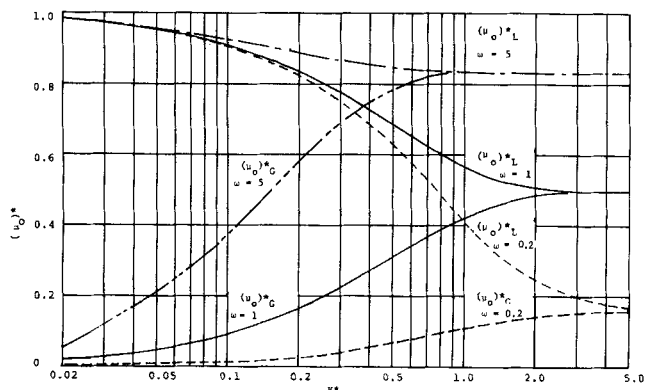


Figure 1. Influence of the overall mass transfer coefficient on the zeroth moment for concurrent flow for a pulse input in the liquid stream.

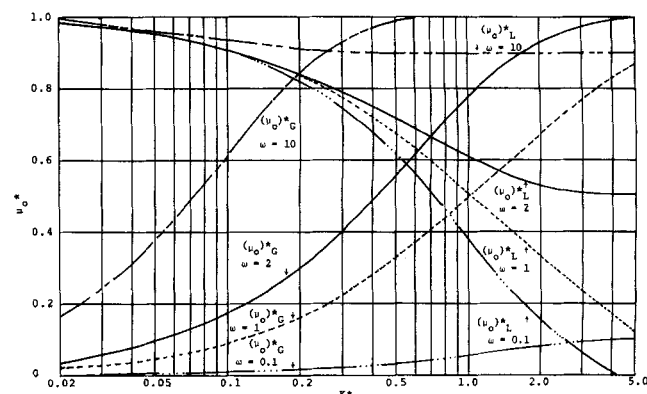


Figure 2. Influence of the overall mass transfer coefficient on the zeroth moment for countercurrent flow for a pulse input in the liquid stream.

The first moment for the liquid phase decreases with increasing K^* and increasing δ . The first moment for the gas phase decreases with increasing δ and exhibits a maximum at a specific value of K^* . Opposite behavior is observed for an input in the gas phase. The complexity of evaluating parameters from the first moment is obvious. The expressions for the first moment subject to the conditions cited above are given in Table 3. From Table 3 it is clear that the first moment now is not equal to the mean residence time of the fluid in a particular phase. Meaningful values for holdups of the various phases can be evaluated reliably if the experiments are performed under such conditions that the asymptotic values of the moments are obtained. For the cases pre-

sented in Figures 3 and 4, as $K^* \rightarrow \infty$, $(\mu_1^*)_L$ and $(\mu_1^*)_G$ approach the asymptote:

$$(\mu_1^*)_L = (\mu_1^*)_G = \frac{\omega^2}{(1+\omega)^2} \left(1 + \frac{1}{\delta} + \frac{1}{\omega\beta} \right) \quad (13)$$

In the case of negligible tracer volatility, $\alpha_1 \rightarrow 0$ and $\alpha_3 \rightarrow 0$; thus $K^* \rightarrow 0$, and the expression for the first moment reduces to the one for the system with a single flowing phase and a stagnant second phase (solid):

$$(\mu_1^*)_L = 1 + \frac{1}{\delta} \quad (14a)$$

$$(\mu_1^*)_G \rightarrow 0 \quad (14b)$$

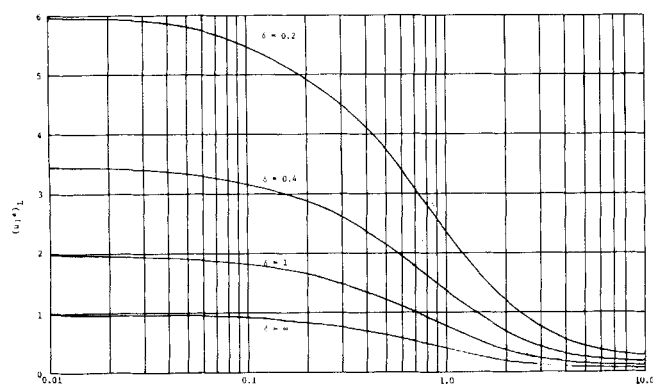


Figure 3. Effect of the overall mass transfer coefficient on the first moment for concurrent flow with a pulse input in the liquid stream ($\omega = 0.2$, $\beta = 20$).

The difference between expressions (13) and (14a) indicate the maximum effect that a tracer volatility and exchange with gas phase may have on the first moment in the liquid phase. The moment for nonvolatile tracers is given by Equation (14a), while for highly volatile tracers it would approach Equation (13). This result was reported by Schwartz et al. (1976). Any tracer experiments in systems with two-phase flow must account for tracer volatility. The existence of the maxima for the first moment in the phase in which the injection did not occur indicates that more accurate measurements may be done at high mass transfer rates by determining moments in the phase in which tracer input was not made.

The expressions for the first moments for the case where the flowing streams are in countercurrent flow are summarized in Table 3. These are subject to the same restrictions as those given for the concurrent flow

TABLE 3. EXPRESSIONS FOR THE FIRST MOMENT FOR TWO-PHASE CONCURRENT FLOW IN PACKED BEDS (NO GAS-SOLID TRANSFER)

1. Concurrent flow (for $f_1 = 1$, $f_2 = 0$)

$$(\mu_1^*)_L = (\mu_0^*)_L \frac{1-\omega}{(1+\omega)^2 K^*} \left(1 + \frac{1}{\delta} - \frac{1}{\beta} \right) - \frac{1}{(1+\omega)^2 K^*} \left\{ \left[\left(1 + \frac{1}{\delta} - \frac{1}{\beta} \right) - K^* \left(1 + \frac{1}{\delta} + \frac{\omega}{\beta} \right) \right] e^{-(1+\omega)K^*} - \left[\omega \left(1 + \frac{1}{\delta} - \frac{1}{\beta} \right) - \omega^2(1+\omega)K^* \right] \right\}$$

$$(\mu_1^*)_G = (\mu_0^*)_G \frac{1-\omega}{(1+\omega)^2 K^*} \left(1 + \frac{1}{\delta} - \frac{1}{\beta} \right) - \frac{\omega}{(1+\omega)^2} \left\{ \left(1 + \frac{1}{\delta} + \frac{\omega}{\beta} \right) e^{-(1+\omega)K^*} - \omega \left(1 + \frac{1}{\delta} + \frac{1}{\omega\beta} \right) \right\}$$

2. Countercurrent flow (for $f_1 = 1$, $f_2 = 0$)

$$(\mu_1^*)_L = \left\{ \frac{\left\{ \left[\frac{1}{\alpha_1} \left(1 + \frac{1}{\delta} \right) + \left(\frac{1}{\alpha_1} + \omega \right) X \right] e^{(\omega-1)\alpha_1} - \left[\left(\frac{1}{\alpha_1} + 1 \right) Y + \frac{1}{\alpha_1} \left(1 + \frac{1}{\delta} \right) \right] \right\}}{\omega e^{(\omega-1)\alpha_1} - 1} - \frac{1}{\alpha_1} \left[1 + \frac{1}{\delta} + (1 + \omega\alpha_1)X \right] \right\} \frac{(1 - e^{(\omega-1)\alpha_1}) e^{(\omega-1)\alpha_1}}{\omega e^{(\omega-1)\alpha_1} - 1} + \frac{X - \omega Y}{\omega e^{(\omega-1)\alpha_1} - 1} e^{(\omega-1)\alpha_1}$$

$$(\mu_1^*)_G = \left\{ \frac{\left\{ \left[\frac{1}{\alpha_1} \left(1 + \frac{1}{\delta} \right) + \left(\frac{1}{\alpha_1} + \omega \right) X \right] e^{(\omega-1)\alpha_1} - \left[\left(\frac{1}{\alpha_1} + 1 \right) Y + \frac{1}{\alpha_1} \left(1 + \frac{1}{\delta} \right) \right] \right\}}{\omega e^{(\omega-1)\alpha_1} - 1} - \frac{1}{\alpha_1} \left[1 + \frac{1}{\delta} + (1 + \omega\alpha_1)X \right] \right\} \frac{1-\omega}{\omega e^{(\omega-1)\alpha_1} - 1} + \frac{X - Y}{\alpha_1(\omega e^{(\omega-1)\alpha_1} - 1)} - \frac{1}{\alpha_1} \left(1 + \frac{1}{\delta} + Y \right)$$

where

$$X = \frac{1 + \frac{1}{\delta} + \frac{\omega}{\beta}}{\omega - 1}; \quad Y = \frac{1}{1 - \omega} \left[\omega \left(1 + \frac{1}{\delta} \right) + \frac{1}{\beta} \right]$$

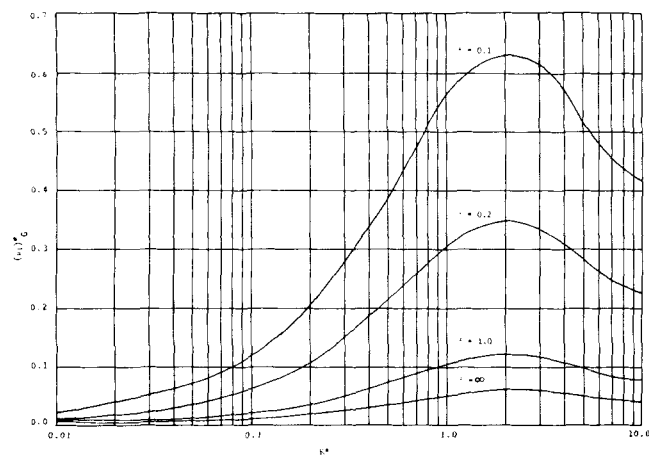


Figure 4. Effect of the overall mass transfer coefficient on the first moment of the gas stream for a pulse input in the liquid stream ($\omega = 0.2$, $\beta = 20$; concurrent flow).

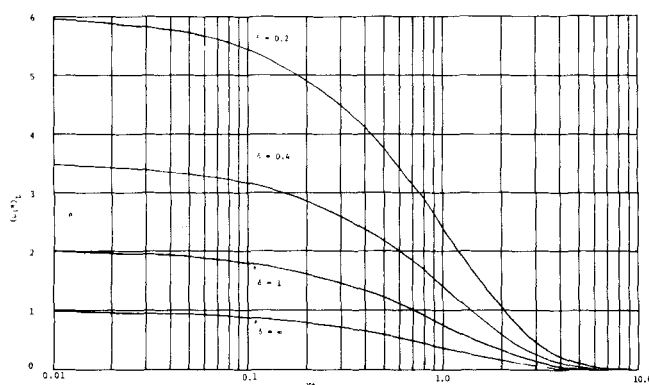


Figure 5. Effect of the overall mass transfer coefficient on the first moment for countercurrent flow with a pulse input in the liquid stream ($\omega = 0.2$, $\beta = 20$; countercurrent flow).

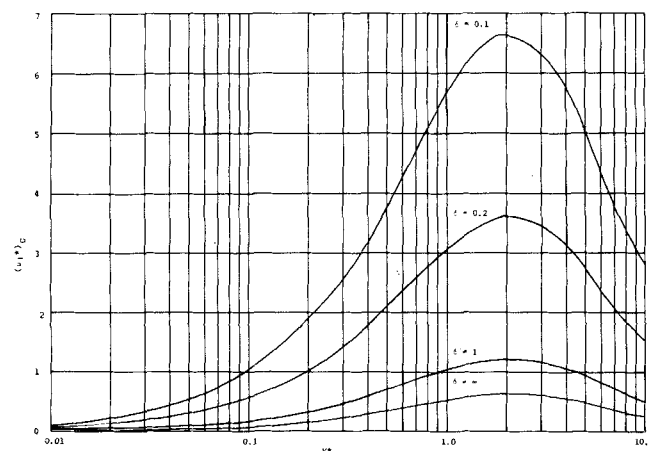


Figure 6. Effect of the overall mass transfer coefficient on the first moment of the gas stream for a pulse input in the liquid stream ($\omega = 0.2$, $\beta = 20$; countercurrent flow).

case and are presented in Figures 5 and 6 corresponding to the liquid and gas, respectively. In either case, the behavior is similar to that for the concurrent flow situation. However, a closer examination of Figure 5 indicates that with increasing K^* and a selected value of δ , the liquid moment decreases more rapidly at large K^* when compared to the results for concurrent flow. This is due to the asymptotic behavior of the first moment as being $(\mu_1^*)_L \sim 0$ as $K^* \rightarrow \infty$, while for cocurrent flow expression (13) results in a finite value. In the case of neg-

ligible tracer volatility ($\alpha_1 \rightarrow 0$), the asymptotic behaviors of both the liquid and gas phase first moments are identical to those given for concurrent flow by expressions (14a) and (14b), respectively, as expected. A careful comparison of Figures 4 and 6 shows that the latter decreases more rapidly with increasing K^* beyond the maximum. This is due to the asymptotic behavior of the corresponding first moment expression being that $(\mu_1^*)_G \sim 0$ for $K^* \rightarrow \infty$ when $\omega < 1$, while that for concurrent flow approaches the finite value given by Equation (13).

CONCLUSIONS

It was shown that the expressions for the moments of tracer impulse responses in two-phase flow through packed beds differ considerably from the ones for single-phase flow through such beds. Expressions for moments in single-phase flow cannot be used in evaluation of holdup, active volume, backmixing, or diffusion parameters unless it is shown that tracer volatility is extremely low, that is, that the tracer does not distribute itself to any appreciable extent into the other flowing phase. The zeroth moment can be used to estimate the mass transfer between the two flowing phases. Holdups of the flowing phases and the amount of the solid contacted can be determined reliably from the expressions for the first moment only under two conditions: if the tracer remains confined in one flowing phase, and if the tracer equilibrates rapidly in both flowing phases. The first moment in the flowing phase into which the impulse injection of tracer was not made shows a maximum at a certain dimensionless mass transfer rate. This behavior is unique to systems with two flowing phases.

ACKNOWLEDGMENT

Partial financial support of this work by the National Science Foundation (Grant No. ENG 73: 08284-AO2) is gratefully acknowledged.

NOTATION

- a_{GS} = gas-solid mass transfer area per unit column volume, cm^2/cm^3
- a_{LG} = gas-liquid mass transfer area per unit column volume, cm^2/cm^3
- a_{LS} = effective liquid-solid particle mass transfer area per unit column volume, cm^2/cm^3
- a = expression defined in Table 1
- A = cross-sectional area of the empty column, cm^2
- b = expression defined in Table 1
- C_L = concentration of the tracer in the liquid, $\text{g-mole}/\text{cm}^3$
- $C_{L,0}$ = input pulse strength, ratio of mass of tracer injected to liquid volumetric flow rate M_T/Q_L , $\text{g-mole s}/\text{cm}^3$
- \overline{C}_L = Laplace transformation C_L , $\text{g-mole}/\text{cm}^3$
- C_G = concentration of the tracer in the vapor or gas, $\text{g-mole s}/\text{cm}^3$
- \overline{C}_G = Laplace transformation of C_G , $\text{g-mole s}/\text{cm}^3$
- C_S = concentration of the tracer on the solid, $\text{g-mole s}/\text{g}$
- \overline{C}_S = Laplace transformation of C_S , $\text{g-mole s}/\text{g}$
- d = expression defined in Table 1
- f = fraction of the input disturbance which is liquid, dimensionless
- f_1 = f
- f_2 = fraction of the input disturbance which is gas or vapor, $\omega(1 - f)$, dimensionless

H_G = gas holdup, volume of gas per unit volume of empty column
 H_L = liquid holdup, volume of liquid per unit volume of empty column
 k_{GS} = gas-solid mass transfer coefficient, cm/s
 k_{LG} = overall gas-liquid mass transfer coefficient, cm/s
 k_{LS} = liquid-solid mass transfer coefficient, cm/s
 K_{GS} = gas-solid adsorption equilibrium constant, cm³/g
 K_{LG} = Henry's law solubility constant between the gas and liquid, C_G/C_L
 K_{LS} = liquid-solid adsorption equilibrium constant, cm³/g
 K^* = overall mass transfer constant defined in Equation (10a), dimensionless
 L = height or length of the packed section, cm
 M_T = total amount of tracer injected in a pulse disturbance, g-mole
 Q_G = volumetric flow rate of the gas or vapor stream, cm³/s
 Q_L = volumetric flow rate of the liquid stream, cm³/s
 s = Laplace transformation variable
 t = time, s
 u_G = superficial gas velocity, cm/s
 u_L = superficial liquid velocity, cm/s
 X = variable defined in Table 2
 Y = variable defined in Table 2
 Z = axial distance in the column, cm

Greek Letters

α_1 = dimensionless gas-liquid mass transfer coefficient, $k_{LGA}L/u_L$
 α_2 = dimensionless liquid-solid mass transfer coefficient, $k_{LSA}L/u_L$
 α_3 = dimensionless gas-solid mass transfer coefficient, $k_{GSA}L/u_L$
 β = dimensionless mean residence time ratio of the liquid to gas defined in Equation (12a) $H_L u_G / u_L H_G = \gamma / \omega$
 γ = $H_L / H_G K_{LG}$
 δ = dimensionless liquid-solid adsorption equilibrium constant defined in Equation (12b) $H_L / (1 - H_L - H_G) K_{LS}$
 $\delta(t)$ = delta function
 λ_1 = expression defined in Table 1
 λ_2 = expression defined in Table 1
 λ_1' = expression defined in Table 1
 λ_2' = expression defined in Table 1
 ω = $u_L / u_G K_{LG}$, defined in Equation (10b)
 $(\mu_n)_G$ = n^{th} moment of the gas effluent, s ^{n}
 $(\mu_n)_L$ = n^{th} moment of the liquid effluent, s ^{n}
 $(\mu_n^*)_L$ = n^{th} reduced moment of the liquid effluent

$(\mu_n^*)_G$ = n^{th} reduced moment of the gas effluent
 $(\mu_0^*)_L$ = zeroth reduced moment of the liquid effluent
 $(\mu_0^*)_G$ = zeroth reduced moment of the gas effluent

Subscripts

G = gas phase
 L = liquid phase
 S = solid phase
 n = moment number

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Manuscript received September 7, 1978; revision received November 20, and accepted January 11, 1979.

Glucose Production by Biochemical Hydrolysis of Mesquite

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The production of glucose from cellulose by enzymatic hydrolysis has been demonstrated by many researchers

0001-1541/79-1028-0890-\$00.75. © The American Institute of Chemical Engineers, 1979.

(Gaden et al., 1976; Millet et al., 1975; Mandels et al., 1974). Enzymatic hydrolysis of cellulose offers a method to convert inexpensive sources of cellulose to glucose